Complex Permittivity Extraction of Vacuum Windows for a Compact FEL-THz Source

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***Abstract*—Exact calculation of the power transmittance requires optical parameters of terahertz (THz) vacuum windows, widely used in the optical path in a vacuum electron system, such as the free electron laser (FEL) THz source. A method for extracting optical parameters of thick vacuum window materials based on THz-TDS was utilized for the determination of emitted power from the down-stream mirror of a compact FEL-THz facility at Huazhong University of Science and Technology (HUST). The real and imaginary part of permittivity for TPX and HRSi were extracted in a wide frequency range from 0.5 to 4.5 THz by the transmission-mode TDS measurement.**

I. INTRODUCTION

In the process of THz transmission in the vacuum environment, the accurate measurement of the complex permittivity of the vacuum window material guarantees better estimation of the power reflection and absorption in the measurement system. The knowledge of material properties can also provide the theoretical and experimental basis for the estimation of absolute power of the emitted THz signal, such as from the resonant cavity in a free electron laser (FEL) THz source [1-3]. The commonly used THz window material in the vacuum environment should have good light transmittance and high hardness. The transmission properties of two types of THz window materials (TPX and HRSi) with a diameter of 2 inches were measured via the TDS technique. An iteration code was written to extract their optical constants, namely the refractive index, extinction coefficient and complex permittivity, with the consideration of the thickness variation. Material properties could be found by minimizing the difference between the measured and modelled complex transmission coefficient in function of the frequency. To achieve the optimization goal, the appropriate fitness function was selected to minimize the difference between the amplitude and phase of the error function at all frequency points. The material properties were extracted between 500 GHz and 4.5 THz, which was selected according to the amplitude level of the Fourier transformed THz signal in the frequency domain.

II. METHODS

For the case of normal incidence of a plane wave on a homogeneous material with thickness *l*, the theoretical complex transmission coefficient is expressed by

$$T(\omega)=\frac{2\mathrm{e}^{-\mathrm{j}(n-1)\omega l/c}}{(\frac{n}{2}+\frac{1}{2n}+1)-(\frac{n}{2}+\frac{1}{2n}-1)\mathrm{e}^{-\mathrm{j}2n\omega l/c}} \qquad (1)$$

where n is the complex refractive index and it is related to the complex relative permittivity by $n^2=\varepsilon_r=\varepsilon'-\mathrm{j}\varepsilon''$. Depending on the material thickness, the transmission multiples due to reflections at the air/sample surface will probably be observed within the measured time window. The measured average thickness is 3.52 mm for TPX and 1.91 mm for HRSi. The refractive index of air is assumed to be unity. The calculated time interval between the first multiple and the main pulse is about 34 ps for TPX and 44 ps for HRSi. At each frequency point, an error function $F(\omega)$ is defined by the absolute difference of amplitude and phase between the measured $T_m(\omega)$ and simulated $T_s(\omega)$ transmission coefficient,

$$F(\omega)=\left\|T_m(\omega)\right|-\left|T_s(\omega)\right|+\left|\angle T_m(\omega)-\angle T_s(\omega)\right\| \qquad (2)$$

For each sample with known thickness, the material properties to be extracted are the real part and the imaginary part of the complex permittivity at each frequency. The initial value of the refractive index is estimated by the time difference between the sample and reference signals. The iteration technique used in this work is the Nelder-Mead simplex method[4,5]. Simplex method is an unconstrained direct search multivariable optimization algorithm. In this problem, when two variables are to be optimized, the regular simplex forms a triangle and the fitness function is calculated at each vertex. Vertices with the highest (or worst) fitness function values are updated with new values according to the rules defined in Ref.[4]. Once the amplitude of fitness function or the standard deviation of the fitness function values at all vertices reaches the threshold, given by the user set at the beginning of the optimization, the optimization finishes. Finally, the corresponding optical constants are extracted in function of the frequency as well as the modelled optimal transmission coefficient with minimum error.

III. RESULTS

In the TDS measurement system, two optical fiber lasers (pulse duration 50 fs, center wavelength 1.55 μm, repetition frequency 50 MHz, average power 20 mW) were used for THz generation and detection by photoconductive antenna. The time and frequency resolution of the generated signal is 2 fs and 7.6 GHz respectively. The measurement without sample was taken as the reference.

Fig.1(a) shows the measured and calculated transmission coefficient of TPX and HRSi from 0.5-4.5 THz after final optimization. The data become noisy at the lower and higher ends of the frequency band due to the relatively weak signal generated by the THz source. The measured and calculated transmission amplitudes agree well and are indistinguishable in the plot. The transmission amplitude decreases with the frequency and the 3.52-mm TPX window has better power transmittance than the 1.91-mm HRSi window. The error function at each frequency (Fig.1(b)) is below 10^{-8} , which indicates that the extracted optical parameters are reliable. The convergence criterial is the same for the two materials.

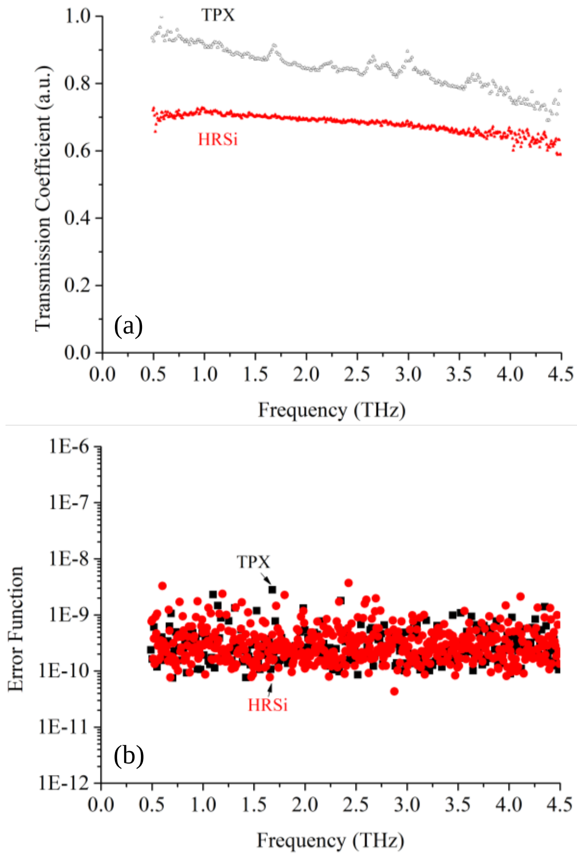

Fig. 1. (a) The measured and modelled amplitude of the transmission coefficient of TPX and HRSi, and (b) the error function.

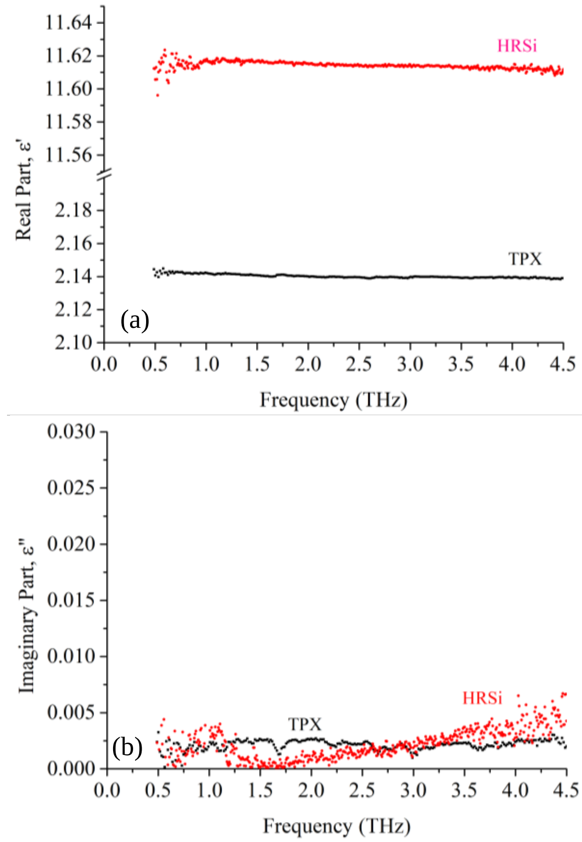

Fig. 2. The extracted optical parameters of TPX and HRSi: (a) real part, and (b) imaginary part of complex permittivity.

Fig.2 shows the extracted real and imaginary part of complex permittivity for the two materials. The optical constants (refractive index and extinction coefficient) could also be obtained (not shown here). The real part of permittivity decrease slightly with the frequency arising from the weak dispersion characteristics for the two materials. At 2 THz, the real part of permittivity is 2.140034 for TPX and 11.615223 for HRSi. Through the comparison with existing values reported in the literature at certain discrete frequency points, the relative difference for the refractive index is below 4% [6].

The dependency of the real part of complex permittivity could be fitted by a second-order polynomial,

$$\varepsilon'_{TPX}(f) = 2.14288 - 0.00142 f + 1.12625 \times 10^{-4} f^2$$

$$\varepsilon'_{HRSi}(f) = 11.61537 + 3.95082 \times 10^{-4} f - 2.95723 \times 10^{-4} f^2$$

where the unit for the frequency is THz.

The loss properties of the two window materials are shown in Fig.2(b). In the studied frequency range, the two materials have the comparable imaginary part of the complex permittivity and its value is below 0.005. For TPX, ε'' is quasi-constant, while for HRSi, ε'' increases slightly with frequency.

The optimization method could be applied in the study of optical constants for other non-absorption materials from transmission-mode TDS measurement technique. The extracted frequency dependent complex permittivity will be helpful in the future calculation of emitted THz power from the FEL-THz source, and it will also be useful in the modeling of electromagnetic properties of materials.

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