Complex Permittivity Reconstruction for Multi-layered Object by Low Complexity Contrast Source Inversion Method

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***Abstract*—This paper introduces complex permittivity reconstruction scheme for multi-layer structured object by contrast source inversion (CSI) scheme in Terahertz(THz) frequency. The inherent problem in CSI based optimization scheme is that the reconstruction accuracy is highly dependent on initial estimates, or number of unknowns. This paper introduces a new optimization scheme by specifying a multi-layer object and exploiting its homogeneity of each layer. The FDTD numerical test demonstrates that our method could provide dielectric property for each layer with considerably lower complexity, that contributes a quantitative material analysis in THz frequency.**

I. Introduction

Terahertz (THz) imaging systems have a great potential for various applications, $e.g.$, non-destructive inspection, security screening, or medical imaging modality. The traditional THz imaging approaches are based on the mapping of reflection strength, which could not offer a quantitative analysis for dielectric property. The dielectric profile reconstruction problem has been intensively studied at the microwave or millimeter wave field, which is well known as inverse scattering analysis. Note that, if each complex permittivity of the multi-layer structure can be quantitatively identified, it directly estimates an absorption spectra for each material, which contributes chemical or physical analysis in the THz band. However, the above inverse problem is, in general, a non-linear and ill-posed problem, hence there are various reconstruction approaches, based on Born approximation. In this paper, we focus on the contrast source inversion (CSI) method [1], which provides accurate dielectric profile without using computationally expensive forward solver (e.g. FDTD method). However, the reconstruction accuracy of the CSI is highly dependent on initial estimates, or the balance between the number of data and unknowns. Focusing on multi-layer analysis with homogeneous media, the proposed method remarkably reduces the dielectric property. The number of unknowns, which could avoid a local optimum problem. Two-dimentional(2-D)FDTD numerical test demonstrates that our proposed method provides accurate dielectric profile for each layer with considerably lower complexity.

II. System Model

Fig **1** shows the observation model. A set of a transmitter and receiver is scanned, or the array with multiple transmitter and receivers are linearly arranged along x axis. It assumes that background media is vacuum, and multi-layer object has a homogeneous dielectric profile at each layer. The i-th layer of object has thickness d_i and complex permittivity ϵ_i at the specific angular frequency ω. Here, a position and thickness of each layer is given.

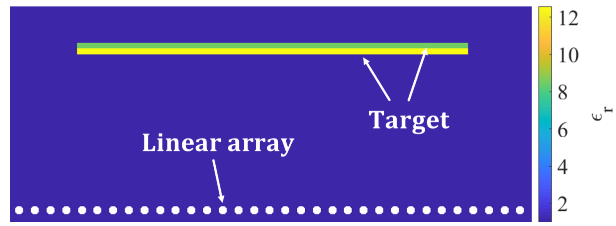

Fig 1. Observation model

III. Method

In assuming that objects exist in the domain $\boldsymbol{r} \in D$, the electric scattered field at the receiver position $\boldsymbol{r}_r$ from the source $\boldsymbol{r}_t$ is expressed by the Helmholtz type domain integral equation as:

$$\begin{aligned} E^{\mathrm{S}}(\boldsymbol{r}_t, \boldsymbol{r}_r) \equiv& E^{\mathrm{T}}(\boldsymbol{r}_t, \boldsymbol{r}_r) - E^{\mathrm{I}}(\boldsymbol{r}_t, \boldsymbol{r}_r) \\ =& (k^{\mathrm{B}})^2 \int_D G^{\mathrm{B}}(\boldsymbol{r}_{\boldsymbol{r}}, \boldsymbol{r}) w(\boldsymbol{r}_t, \boldsymbol{r}) d\boldsymbol{r}, \end{aligned} \tag{1}$$

where $E^{\mathrm{T}}(\boldsymbol{r}_t, \boldsymbol{r}_r)$ and $E^{\mathrm{I}}(\boldsymbol{r}_t, \boldsymbol{r}_r)$ denote total and incident electric fields, respectively, k^{B} is wavenumber for the background. The domains D and S denote the region of interest (ROI) and that including the source and observation points, $G^{\mathrm{B}}(\boldsymbol{r}_r, \boldsymbol{r})$ is Green's function of the background, $w(\boldsymbol{r}_t, \boldsymbol{r}) \equiv E^{\mathrm{T}}(\boldsymbol{r}_t, \boldsymbol{r}_r)\chi(\boldsymbol{r})$ is named as contrast source, where $\chi(\boldsymbol{r}) \equiv \epsilon(\boldsymbol{r})/\epsilon^B(\boldsymbol{r}) - 1$ denotes a contrast function, corresponds to complex permittivity. The CSI solves the optimal χ by minimizing the following cost function:

$$\begin{aligned} F(\chi, w) \equiv& \frac{\sum_{r_t} \|E^{\mathrm{S}}(\boldsymbol{r}_t, \boldsymbol{r}_r) - \mathcal{G}^S[w]\|_S^2}{\sum_j \|E^{\mathrm{S}}(\boldsymbol{r}_t, \boldsymbol{r}_r)\|_S^2} \\ &+ \frac{\sum_{r_t} \|\chi(\boldsymbol{r}) E^{\mathrm{I}}(\boldsymbol{r}_t, \boldsymbol{r}') - w(\boldsymbol{r}_t, \boldsymbol{r}) + \chi(\boldsymbol{r})\mathcal{G}^D[w]\|_D^2}{\sum_{r_t} \|\chi(\boldsymbol{r}) E^{\mathrm{I}}(\boldsymbol{r}_t, \boldsymbol{r}')\|_D^2}, \end{aligned} \tag{2}$$

Here, the following notations are defined as:

$$\mathcal{G}^S[w] = (k^B)^2 \int_D G^B(\boldsymbol{r}_r, \boldsymbol{r}) w(\boldsymbol{r}_t, \boldsymbol{r}) d\boldsymbol{r}, (\boldsymbol{r} \in D), \tag{3}$$

$$\mathcal{G}^D[w] = (k^B)^2 \int_D G^B(\boldsymbol{r}', \boldsymbol{r}) w(\boldsymbol{r}_t, \boldsymbol{r}) d\boldsymbol{r}, (\boldsymbol{r}' \in D), \tag{4}$$

where $\|\cdot\|_S^2$ and $\|\cdot\|_D^2$ are l_2 norms defined as the S and D, respectively. The cost function $F(\chi, w)$ expresses the sum of the two type of domain integral equation in Eq. (1) for both the domain S and D. Note that, the above cost function is minimized by sequentially updating the valuables as $w(\boldsymbol{r}_t, \boldsymbol{r})$, $E^{\mathrm{T}}(\boldsymbol{r}_t, \boldsymbol{r}_r)$, and $\chi(\boldsymbol{r})$, namely, there is no need for calculating the total field in the domain of D, which avoids a large amount of computational cost, compared with other inverse scattering analyses. However, the original CSI suffers from inaccuracy due to ill-posed conditions, that the number of unknowns are considerably larger than that of data. To avoid the above, the proposed method introduces a sizable reduction of the number of unknowns by introducing that each layer has a homogeneous media, and simplifying the variables as $\boldsymbol{\epsilon} \equiv (\epsilon_1, ..., \epsilon_n)$, where ϵ_n denotes the complex permittivity of the n-th layer.

IV. Results in Numerical Simulation

The 2-D FDTD numerical test with 20 μm cell size is investigated as follows, where the part of layer properties are referred from Qaddoumi *et al* [2]. Background medium is sets as vacuum, and the complex relative permittivity of the first layer is $12.58 + 1.31j$ and that of the second layer is $8.42 + 0.57j$. 33 set of transmitter and receiver are linearly arranged with 0.16 mm equally spacing on $y = 0$, and those all combinations are used for the inversion scheme. The transmitted pulse forms the Gaussian modulated pulse with 0.24 THz center frequency and 0.27 THz bandwidth. The CSI inversion is done by using the single frequency data as 0.20 THz. Fig **1** indicates numerical simulation model. The maximal iteration number of the CSI [3] is set to 10000. The total number of unknowns is 1800 in the original CSI, while that of data is 33 in this case. Here, the conductivity for each layer is given at the proposed method, for simplicity.

Fig **2** show the reconstruction results using the original CSI and the proposed methods. As shown in this figure, the original CSI suffers from the inaccuracy for complex permittivity reconstruction, especially for both ends of each layer, while the proposed method accurately provides relative permittivity for each layer. Here, the root mean square error (RMSE) for the real complex permittivity is introduced for the quantitative error analysis. The RMSE and calculation times are summarized in Tab I, using Intel(R) Xeon(R) Gold 6130CPU 2.10GHz processor and 704GB RAM. These quantitative evaluation demonstrated that our method achieves more accurate and faster reconstruction of dielectric profile, even in dealing with a considerably large number of unknowns. Fig **3** show the minimized cost function results for each permittivity in the proposed method, and this figure denotes that our proposed approach could calculate an appropriate set of relative permittivity as (ϵ_1, ϵ_2) by investigating the cost function of each combination.

V. Conclusion

This paper proposed the CSI based complex permittivity reconstruction scheme for the THz-band multi-layer analysis problem, where the global search using the cost function could avoid the local optimal issue. It is our current work to validate this method through the experimental data, such as THz-TDS data.

TABLE I: Comparison of RMSEs and calculation times of each method.

Method	Original CSI	Proposed method
RMSE[%]	1.85×10^2	2.75×10
Calculation time[s]	1.53×10^4	5.15×10^3

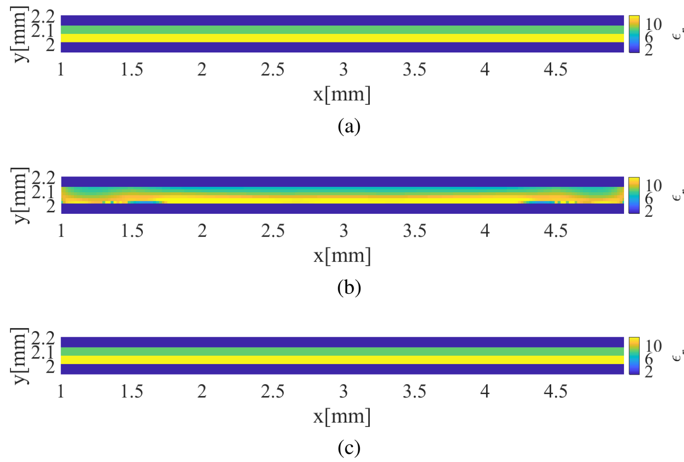

Fig 2. (a): Reconstruction results of real part of complex permittivity. (a): Original profile. (b): Original CSI. (c): Proposed method. Color denotes real part of permittivity.

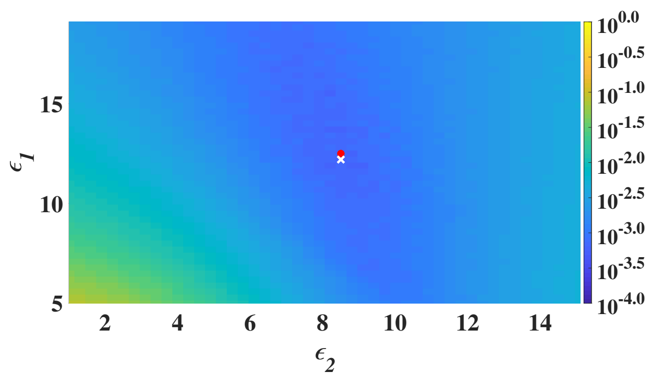

Fig 3. Minimal values of the cost function of the CSI at each combination of (ϵ_1, ϵ_2), where the original combination is denoted as red solid circle, estimation is denoted as white cross. Color denotes the residual of the cost function.

References

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