The structure of azimuthal eigenmodes of the optical-terahertz biphoton field

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***Abstract*—The angular structure of an optical-terahertz biphoton field generated by strongly non-degenerate parametric down-conversion is studied theoretically. Azimuthal eigenmodes of scattered radiation are found, in the basis of which the nonlinear interaction operator has a diagonal form. The solution of the evolution equations for these eigenmodes allows us to find a scattering matrix describing the intensity and correlation properties of nonclassical optical-terahertz scattered radiation at arbitrary parametric gain and at different frequencies of the idler terahertz wave.**

I. INTRODUCTION

QUANTUM correlated optical-terahertz biphoton states are created in the process of strongly non-degenerate parametric down-conversion (PDC). Terahertz radiation obtained in this way can be used for terahertz spectroscopy [1], quantum sensing and imaging [2] as well as for measuring the absolute brightness of terahertz radiation [3]. The possibility of using quantum-optical technologies in the terahertz frequency range is also of great interest. However, quantum correlations between optical signal and idler terahertz photons have not yet been registered [4]. This is due to the absence of single-photon terahertz detectors, as well as to the large number of entangled angular modes in the generated radiation. The method developed in this work makes it possible to obtain the mode structure of macroscopic quantum states of an optical-terahertz field generated by stimulated PDC, which can be registered by modern terahertz detectors.

II. OPERATOR OF NONLINEAR INTERACTION

To analyze the angular structure of the optical-terahertz biphoton field, we study the nonlinear interaction operator describing PDC. It can be shown that in the case of strongly non-degenerate PDC, when the wavelength of the terahertz idler field is comparable to the pump beam width d, the main entanglement in the scattered radiation is associated with azimuthal angles φ_i, φ_s. Then, in the directions of exact phase matching in polar angles θ_i, θ_s, the nonlinear interaction operator for strongly non-degenerate PDC under certain approximations takes the following form:

$$\hat{G}_{nl} \approx \hbar\gamma \left\{ \int_0^{2\pi} \chi_{eff}^{(2)}(\varphi_s, \varphi_i) e^{-\tau\{1+\cos(\varphi_i-\varphi_s)\}} \hat{a}_i^+ \hat{a}_s^+ d\varphi_s d\varphi_i \right\} + h.c.,$$

where $\tau = d^2 k_i k_s \sin\theta_i \sin\theta_s / 2$ is a control parameter. By a series of transformations, this nonlinear interaction operator can be reduced to the diagonal form $\hat{G}_{nl} = \hbar\gamma \sum_j R_j \hat{b}_j^+ \hat{c}_j^+ + h.c.$

Here, R_j are its eigenvalues, and the operators of the photon creation in eigenmodes $\hat{b}_j^+$ and $\hat{c}_j^+$ are related to the initial field operators by unitary transformations

$$\hat{b}_j^+ = \int_{-\pi}^{\pi} U_j(\varphi_i) a_i^+(\varphi_i) d\varphi_i; \; \hat{c}_j^+ = \int_{-\pi}^{\pi} U_j(\varphi_s) a_s^+(\varphi_s) d\varphi_s .$$

The solution of the evolution equations for them has the form of Bogolyubov transformations:

$$\begin{cases} \hat{b}_j^+(L) = \hat{b}_j^+(0) \operatorname{ch} g_j + \hat{c}_j(0) \operatorname{sh} g_j \\ \hat{c}_j^+(L) = \hat{b}_j(0) \operatorname{sh} g_j + \hat{c}_j^+(0) \operatorname{ch} g_j \end{cases},$$

and the gain for each mode is proportional to its eigenvalue: $g_j = |R_j|\gamma L$. In the case of a small parametric gain $\gamma L \ll 1$, these eigenmodes correspond to Schmidt modes.

III. AZIMUTAL EIGENMODES

The solution in the basis of eigenmodes allows us to obtain the scattering matrix in the basis of the initial modes of plane waves, which determines the angular spectrum and correlation properties of the optical-terahertz biphoton field. For example, the angular dependence of the scattered radiation intensity has the form $I_j(\varphi_{i,s}) \sim \sum_j \operatorname{sh}^2 g_j |U_j(\varphi_{i,s})|^2$, that is, it is the sum of the intensities of eigenmodes with weights determined by their eigenvalues.

For numerical estimates, as in [4], we consider noncollinear eee-type PDC in a lithium niobate crystal. In this case, the effective quadratic susceptibility can be approximated by the dependence $\chi_{eff}^{(2)}(\varphi_s, \varphi_i) \approx (1+\cos^2\varphi_i)/2$. Fig.1 shows the dependence of the squared eigenvalues R_j^2 on the frequency of the idler terahertz wave. It can be seen that at very low

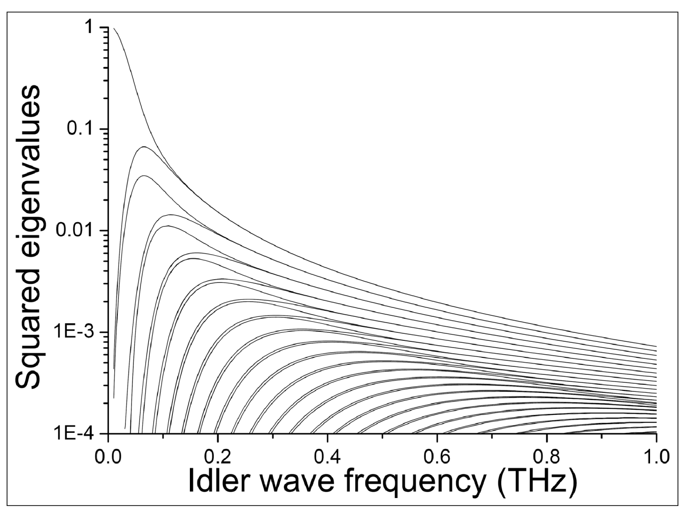

Fig. 1. Dependence of the squared eigenvalues of azimuthal eigenmodes on the terahertz frequency of the idler wave of scattered radiation.

frequencies, almost entire PDC radiation is concentrated in a single mode with the maximum eigenvalue. However, as the frequency increases, the difference between the eigenvalues decreases, and the effective number of modes increases.

Figure 2 shows the angular dependences of various eigenmodes at the idler radiation frequency 0.5 THz. In this case, the effective number of modes is several tens. It can be seen that the modes corresponding to positive and negative eigenvalues are practically the same. This is due to the twofold degeneracy of the eigenvalues at large values of the control parameter $\tau \gg 1$. Note that the estimation of the Fedorov parameter for the effective number of azimuthal modes in [4] is underestimated by two times due to this twofold degeneracy.

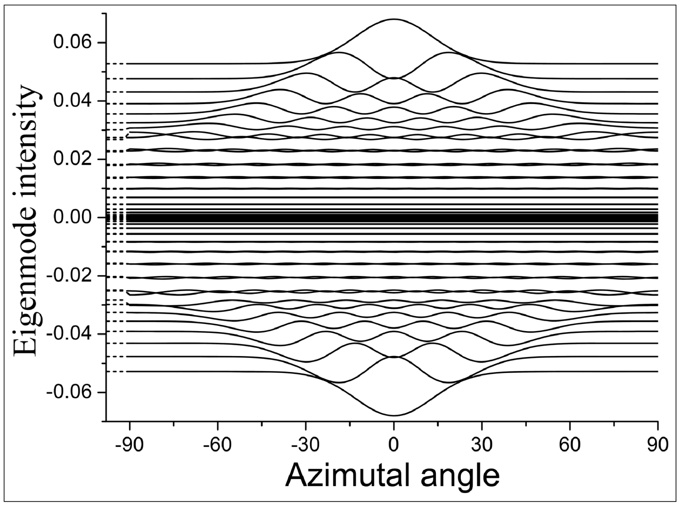

Fig. 2. The angular form of the azimuthal eigenmodes at the idler terahertz radiation frequency 0.5 THz. For clarity, the intensity of each eigenmode is shown relative to the level of its eigenvalue (shown by the dotted line on the left).

IV. Summary

In conclusion, the developed method for obtaining azimuthal eigenmodes at strongly non-degenerate PDC allows one to describe the effect of the mode structure of scattered radiation on the nonclassical properties of the optical-terahertz biphoton field, which can be used to apply quantum information technologies in the terahertz frequency range, for example, to develop a method for calibration the sensitivity of terahertz detectors.

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