Analytical Modeling of Terahertz Time-Domain Spectroscopy with Monolithic Mode-Locked Laser Diodes

Kevin Kolpatzeck[1], Xuan Liu[1], Kai-Henning Tybussek[1], Lars Häring[1], Marlene Zander[2], Wolfgang Rehbein[2], Martin Moehrle[2], Andreas Czylwik[1], and Jan C. Balzer[1]

[1]University of Duisburg-Essen, Bismarckstr. 81, 47057 Duisburg, Germany
[2]Fraunhofer Heinrich-Hertz-Institute (HHI), Einsteinufer 37, 10587 Berlin, Germany

***Abstract*—In this work, we present an analytical model that links the complex terahertz spectrum acquired with a terahertz time-domain spectroscopy (THz-TDS) system to the complex optical spectrum from a monolithic mode-locked laser diode (MLLD) and the transfer function of the terahertz channel. It also gives the relationship between the optical intensity autocorrelation and the terahertz spectrum. This model greatly improves the understanding of THz-TDS with ultra-high repetition rate lasers and enables the design of lasers with optimized optical spectra.**

I. Introduction

ONE major contributor in terms of size, weight, and cost of state of the art THz-TDS systems is the femtosecond fiber laser. A promising alternative as driving source are MLLDs [1,2]. They have the potential to enable compact and low-cost systems and their ultra-high repetition rate eliminates the need for accurate matching of the transmit and receive path lengths, which is especially beneficial for reflection mode measurements. In this work, we improve the understanding of THz-TDS with MLLDs by presenting an analytical model that relates the complex optical spectrum of the MLLD to the complex terahertz spectrum.

II. Modeling

Our model considers a conventional THz-TDS setup consisting of a light source whose output is distributed to a photodiode-based emitter and – through a variable delay line – to a photoconductive receiver. The starting point of the model is the complex optical spectrum of the MLLD [3]:

$$e_{\mathrm{opt}}(t) = \sum_{k=0}^{N-1} E_k \cdot \mathrm{e}^{\mathrm{j}[2\pi(f_0+kF)\cdot t+\varphi_k]}\,, \quad (1)$$

where N denotes the number of considered laser modes, f_0 is the frequency of the first mode, F is the repetition rate of the laser, and E_k and φ_k denote the amplitude and phase of mode k, respectively. Now, the detected terahertz signal at the output of the photoconductive receiver can be calculated as [3]:

$$i_{\mathrm{det}}(\tau) \propto \sum_{m=1}^{N-1} |H_{\mathrm{THz}}(mF)| \cdot \sin[2\pi mF\tau + \angle H_{\mathrm{THz}}(mF)] \cdot A_m\,, \quad (2)$$

where $H_{\mathrm{THz}}(mF)$ is the transfer function of the terahertz channel, τ is the delay of the receiver arm, and the terms A_m are amplitude factors that depend exclusively on the complex optical spectrum through the relationship [3]:

$$A_m = \sum_{k=m}^{N-1}\sum_{l=m}^{N-1} E_k E_{k-m} E_l E_{l-m} \cdot \cos[(\varphi_k - \varphi_{k-m}) - (\varphi_l - \varphi_{l-m})]\,. \quad (3)$$

Similarly, the intensity autocorrelation can be expressed as [3]:

$$R_{pp}(\tau) \propto \sum_{m=0}^{N-1} \cos(2\pi mF\tau) \cdot A_m\,. \quad (4)$$

It can be observed that the terahertz amplitude spectrum is the amplitude spectrum of the intensity autocorrelation weighted with the transfer function of the terahertz channel. Thus, the intensity autocorrelation of a MLLD can predict its performance in a THz-TDS system.

III. Experimental Results

To test the validity of the model, we use a two-section mode-locked quantum-dot (QD) laser diode from Fraunhofer Heinrich-Hertz-Institute (HHI) in a THz-TDS system as depicted in Fig. 1. The MLLD has a repetition rate of 50.17 GHz and is tuned to a central frequency of 192.5 THz [4]. The terahertz emitter is an InGaAs photodiode-based module and the detector is an InGaAs photoconductive antenna-based module, both from Fraunhofer HHI [5]. The polarization of the optical output signal of the MLLD is aligned to the slow axis of the polarization-maintaining fibers in the spectrometer with a polarization controller in tandem with a polarization splitter. We use an OZ Optics ODL-650 motorized variable delay line in the receiver path and a Zurich Instrument MFLI lock-in amplifier for measuring the photocurrent at the output of the terahertz detector.

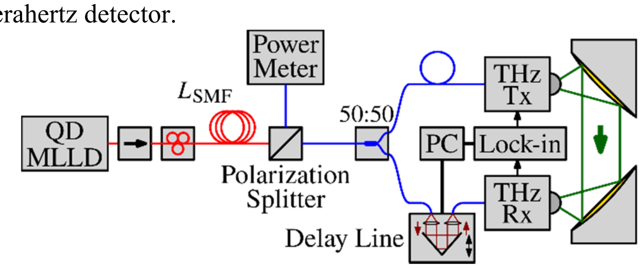

Fig. 1. THz-TDS measurement setup. Red lines represent single-mode optical fibers. Blue lines represent polarization-maintaining optical fibers. Black lines represent electrical signals.

Fig. 2(a) shows the amplitude spectrum of the MLLD measured with an Anritsu MS9740A optical spectrum analyzer. We adjust the laser chirp at the input of the THz spectrometer by using the anomalous dispersion of a variable-length section of single-mode fiber between the polarization controller and the polarization splitter. For the nearly chirp-free case obtained with the single-mode fiber length $L_{\mathrm{SMF}} = 50$ m, Fig. 2(b) shows

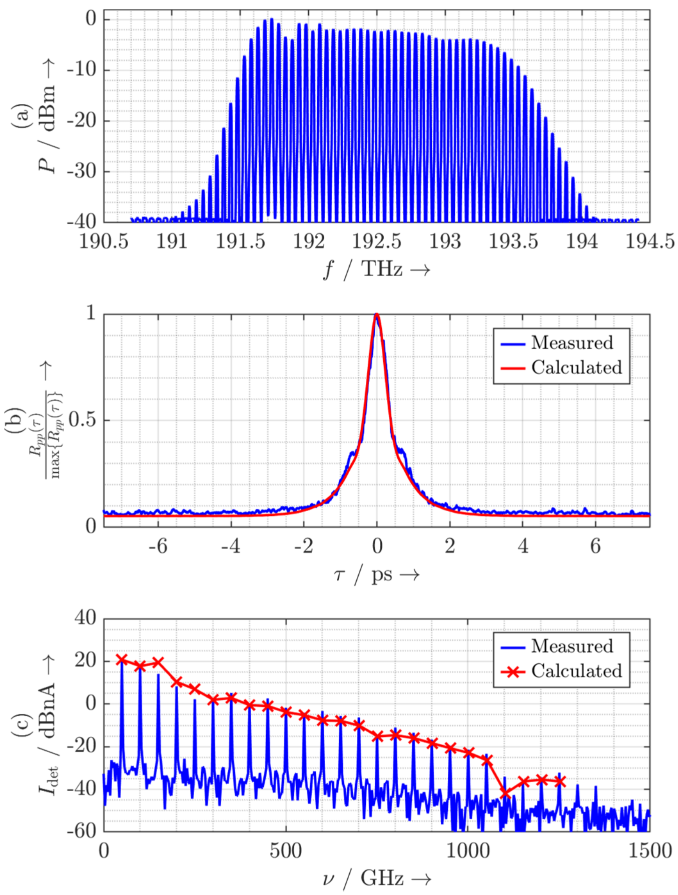

Fig. 2. (a) Measured optical spectrum. (b) Measured and calculated intensity autocorrelation for the chirp-free case. (c) Measured and calculated terahertz spectrum for the chirp-free case.

the intensity autocorrelation measured with an APE pulseCheck second harmonic autocorrelator in blue and the intensity autocorrelation calculated with the equations given above in red. The calculated autocorrelation matches the measured autocorrelation both in terms of shape and pulse width. Fig. 1(c) shows the corresponding measured and calculated terahertz spectra. The calculated spectrum is normalized to the measured spectrum to facilitate the comparison. The amplitudes match remarkably well with the measured terahertz spectrum across the entire detectable frequency range. This clearly validates our proposed analytical model.

IV. Conclusion

We have shown a model that predicts the detected terahertz spectrum of a THz-TDS system driven by a MLLD from the complex optical spectrum of the MLLD and the transfer function of the terahertz path. The model will help to gain a better understanding of THz-TDS systems, especially with ultra-high repetition rates. Further, it enables the systematic development of MLLDs for THz-TDS.

Acknowledgement

This work has been funded by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) - Project-ID 287022738 - TRR 196.

References

[1]. K. Merghem, *et al.*, "Terahertz Time-Domain Spectroscopy System Driven by a Monolithic Semiconductor Laser," *J. Infrared, Millimeter, Terahertz Waves* **38**(8), 958–962 (2017).

[2]. K.-H. Tybussek *et al.*, "Terahertz Time-Domain Spectroscopy Based on Commercially Available 1550 nm Fabry–Perot Laser Diode and ErAs:In(Al)GaAs Photoconductors," *Appl. Sci.* **9**(13), 2704 (2019).

[3]. K. Kolpatzeck *et al.*, "System-theoretical modeling of terahertz time-domain spectroscopy with ultra-high repetition rate mode-locked lasers," *Opt. Express* **2020***, 28*, 16935–16950.

[4]. M. Zander *et al.*, "High performance BH InAs/InP QD and InGaAsP/InP QW mode-locked lasers as comb and pulse sources," in *Optical Fiber Communication Conference (OFC) 2020*, 2020, p. T3C.4,.

[5]. T. Göbel *et al.*, "Telecom technology based continuous wave terahertz photomixing system with 105 decibel signal-to-noise ratio and 3.5 terahertz bandwidth," *Opt. Lett*. **38**(20), 4197–4199 (2013).