Theoretical Investigation into Cherenkov Radiation in an Anisotropic Double-Negative Medium

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***Abstract*—Cherenkov radiation for particles traveling in an unbounded anisotropic double-negative medium is theoretically investigated to obtain the analytical expressions for the Cherenkov radiation condition, Cherenkov radiation angle and inversed Cherenkov radiation. The total radiated energy (spectral density) per unit length of path is derived by using rigorous field theory. As a specific example, the effects of loss of the anisotropic double-negative medium and the effective plasma frequency on the total radiated energy per unit length of path are also discussed. It turns out that the wave vector and time-averaged Poynting vector in an anisotropic double-negative medium are, in generally, not exactly anti-parallel, and the Cherenkov radiation angle and the total radiated energy in this case is different from that in the isotropic double-negative medium.**

I. Introduction And Background

RADIATION is emitted when a charged particle moves in a material with a speed faster than the phase velocity of light in that material. This phenomenon is called Cherenkov radiation (CR)[1,2]. It has been broadly used in high-energy particle physics, optics, cosmic-ray physics and high-power microwave sources.

In recent years, due to the appearance of double-negative medium (DNM), the research on CR has been receiving much attention. CR by a point charge in an infinite isotropic DNM was studied in both lossy and dispersive cases in detail[3]. Besides, CR by an electron bunch that moves in a vacuum above an isotropic DNM was also theoretically investigated[4]. The research results show that the reversed CR[2] is further predicted from the theoretical viewpoint in the isotropic DNM, and the CR detectors based on the use of DNM was proposed, etc. Currently, a few researchers have addressed the physical importance and experimental design of this phenomenon in detail. Since a DNM is in essence an anisotropic medium but not isotropic medium, it is of interest to study the CR in the anisotropic DNM in order to further explore its physical properties and potential applications.

II. Results

We consider an infinite, linear, inhomogeneous, anisotropic and transparent DNM which has a negative permittivity tensor described by Drude model and a negative permeability tensor depicted by Lorentz model, and supposes that a particle moves along the *z-axis* with a constant velocity $\bar{\upsilon} = \hat{z}\upsilon$. CR condition and CR angle in an unbounded anisotropic DNM were obtained by using the macroscopic electromagnetic theory and written respectively as follows.

$$\mathrm{Re}\{\varepsilon_{r\rho}\mu_{r\theta}\} > 1/\beta^2 \ , \tag{1}$$

$$\cos\theta_{CR} = -\frac{1}{\mathrm{Re}\{\sqrt{\varepsilon_z\mu_\theta + (1-\varepsilon_z/\varepsilon_\rho)/\upsilon^2}\}\upsilon} \ . \tag{2}$$

The energy lost by moving charge due to the CR is calculated as follows:

$$\frac{dW}{dz} = qE_z\Big|_{z\to\upsilon t,\,\rho\to 0}$$
$$= \frac{q^2}{4\pi}\mathrm{Re}\left\{\int_0^\infty d\omega\frac{k_\rho}{\varepsilon_z\omega}\left[\frac{1}{\rho_0}H_1^{(2)}(k_\rho\rho_0) + \frac{k_\rho}{2}[H_0^{(2)}(k_\rho\rho_0) - H_2^{(2)}(k_\rho\rho_0)]\right]\right\} \tag{3}$$

where $\varepsilon_\rho, \varepsilon_z$, and μ_θ are the permittivities in the ρ and z directions and permeability in the θ direction respectively, $\beta = \upsilon/c$, c is the light velocity of free space and $\mathrm{Re}\{\ \}$ is the real part operator, k_ρ is the radial wave number, the quantity ρ_0 is the minimum average distance to the field source for which classical electrodynamics still holds, and here we choose $\rho_0 = 1\mu m$ for all frequencies investigated here. Note that the integral interval of the above Formula (3) is, in fact, decided by Formula (1).

First, we study the CR angle θ_{CR} and the relationship between time-averaged Poynting vector $<\bar{S}>$ and wave vector $\bar{k} = -\hat{\rho}\,\mathrm{Re}(k_\rho) + \hat{z}k_z\left(k_z = \omega/\upsilon\right)$. As illustrated in Fig. 1, we can find that θ_{CR} is an obtuse angle, while in the normal medium, θ_{CR} is an acute angle. The directions of the time-averaged Poynting vector $<\bar{S}>$ and wave vector $\bar{k}$ in the anisotropic DNM, generally speaking, are not exactly anti-parallel in Fig. 1. This characteristic is different from that in isotropic DNM, because the vectors $<\bar{S}>$ and $\bar{k}$ in the isotropic DNM are anti-parallel[2,4]. The CR is also reversed, just like that in the isotropic DNM. Same as in the case of the normal media[1], Formula (2) predicts the dependence of the CR angle θ_{CR} on the velocity of the particle and the medium. So the desired CR angle can be easily obtained by changing the particle velocity or the medium. In another case, if $\mathrm{Re}\{\varepsilon_{r\rho}\mu_{r\theta}\} < 1/\beta^2$ then the wave excited by a moving particle is evanescent wave and CR does not happen. Here we do not discuss this case.

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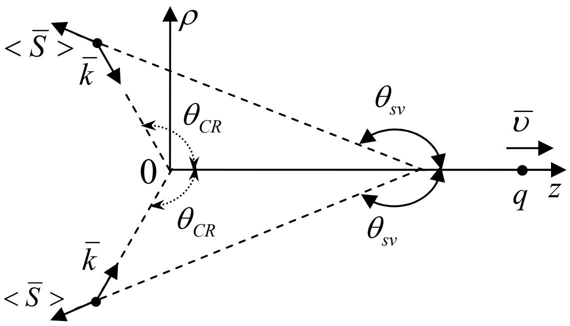

Fig 1. A schematic diagram of reversed Cherenkov radiation in the anisotropic DNM.

Next, we investigate the CR condition. We calculate CR condition shown in Fig. 2. In this Figure, n_1, n_2, and n_3 denote the complex negative refraction index in the DNM corresponding to $\omega_{p\rho} = 2\pi \times 7 \times 10^9$ rad/s, $\omega_{p\rho} = 2\pi \times 10^{10}$ rad/s, $\omega_{p\rho} = 2\pi \times 13 \times 10^9$ rad/s respectively. Here $\omega_{p\rho}$ denotes the effective plasma frequency in the ρ direction. The dashed lines indicate the values of $\mathrm{Re}\{n^2\}$, and the solid line represents $1/\beta^2$, above which the CR condition can be met and CR can happen in an operating frequency band. It is seen that the value of $\mathrm{Re}\{n^2\}$ decreases as the operating frequency increases, and increases with $\omega_{p\rho}$ for a given operating frequency. Clearly, the operating frequency band increases as the $\omega_{p\rho}$ increases. This could be understood as follows. As the $\omega_{p\rho}$ increases the absolute value of $\mathrm{Re}(\varepsilon_\rho)$ increases at a given operating frequency, which means the phase velocity decreases, thus the CR condition can be more easily satisfied.

Finally, to study the effects of the loss and different effective plasma frequency on the total radiated energy, numerical integration is performed. The numerical results are shown in Fig. 3. Note that case 1, case 2, and case 3 refer to $\omega_{pz} < \omega_{p\rho}$, $\omega_{pz} = \omega_{p\rho}$ and $\omega_{pz} > \omega_{p\rho}$ respectively. Here let $\omega_{pz} = 2\pi \times 7 \times 10^9$ rad/s in the case 1, $\omega_{pz} = 2\pi \times 10^{10}$ rad/s in the case 2 and $\omega_{pz} = 2\pi \times 13 \times 10^9$ rad/s in the case 3, where ω_{pz} denotes the effective plasma frequency in the z direction. Clearly, the case 2 implies that the anisotropic DNM reduces to the isotropic DNM. From Fig. 3, it is seen that the total radiated energy increases as the loss decreases for both anisotropic and isotropic DNMs. However, at a certain loss, the total radiated energy in the anisotropic DNM (case 1) is larger than that in isotropic DNM (case 2), while the total radiated energy in the anisotropic DNM (case 3) is smaller than that in isotropic DNM (case 2). At the same time, when the loss increases, the differences of the total radiated energy between the anisotropic and isotropic DNMs become smaller, which is a direct result of the fact that the total radiated energy distributions tend to overlap.

Therefore, the unique properties may be useful for detecting or identifying the particle in particle physics and for wave production in high-power microwave sources.

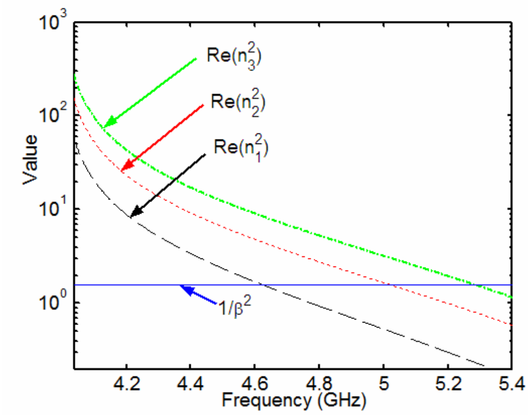

Fig. 2. Frequency dependence of Cherenkov radiation condition for different negative refractive index.

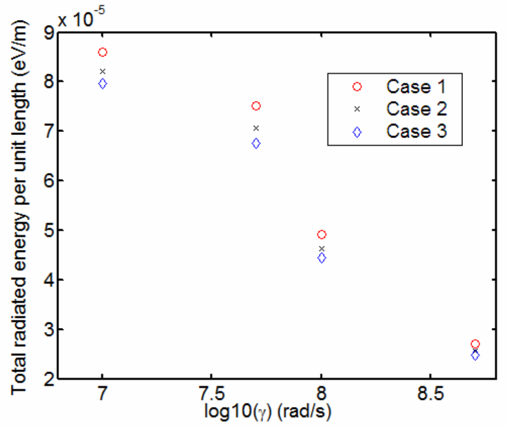

Fig. 3. Comparison of total radiated energy among the three cases for different loss.

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