Free-Electron Laser with Resonant Pumping

Kostyantyn Ilyenko[a], Vitaliy A. Goryashko[a], and Anatoliy Opanasenko[b]
[a]Institute for Radiophysics and Electronics of NASU, vul. Ak. Proskury, 12, Kharkiv, 61085 Ukraine
[b]National Science Center "Kharkiv Institute of Physics and Technology" of NASU, vul. Akademichna, 1, Kharkiv, 61108 Ukraine

***Abstract*—We find the boundaries of regular and chaotic dynamics zones around the magnetoresonance for a hybrid planar free-electron laser amplifier. For the zones of regular dynamics the maximal magnetoresonant gain is calculated analytically and is shown to be in a good agreement with non-linear numerical simulations. It is also analytically demonstrated that in the vicinity of magnetoresonance the gain is independent of the spatial amplitude of undulator magnetic field. Moreover, we find that in the zone of regular dynamics above the magnetoresonance the efficiency of amplification achieves its maximal value and is less prone to the influence of space-charge effects.**

I. Introduction And Background

Modern experimental setups of free-electron lasers with guide longitudinal magnetic field $B_{\parallel}$ (hybrid FELs with pump magnetic field $\vec{B}_p(\vec{r}) = (0, -B_{\perp}\sin(2\pi z/\lambda_w), -B_{\parallel})$) usually operate well below its magnetoresonant value $B_{\parallel}^{res} \approx m_e c\gamma_0\omega_0/|e|$, where $\omega_0 = 2\pi V_{\parallel}/\lambda_w$, m_e, e, γ_0 and c are the undulator frequency, electron rest mass, its charge, relativistic factor and speed of light in vacuum, respectively. However, the potential advantage of magnetoresonant regime was indicated as early as in[1] because of expected increase in the efficiency of beam-wave interaction. The reason for not preferring the magnetoresonant regime lies in the electron beam instability found in numerical simulations and occasionally observed experimentally[2]. In this contribution, we employed the Chirikov resonance-overlap criterion to provide an analytical estimate of the width of the zone of chaotic dynamics of electrons and find the gain in the vicinity of magnetoresonance for a hybrid planar FEL amplifier. Using Kisunko-Vainshtein's equations of excitation of a regular waveguide by an electron beam, the frequency f, gain G, efficiency and saturation length as functions of $B_{\parallel}$ are calculated.

II. Results

Analytical linear in the microwave field theory of beam-wave interaction for a hybrid planar FEL amplifier can be constructed along the same lines as the highly successful theory of gyrotron provided analytical expressions for single-particle trajectories of electrons in the combined pump magnetostatic field are known[3-5]. Solutions to the corresponding non-linear equations are found numerically. To the order $o(\varepsilon^2)$ ($\varepsilon = \omega_{\perp}/\omega_0 \equiv cK/(\gamma_0 V_{\parallel})$, $\omega_{\perp,\parallel} = |e|B_{\perp,\parallel}/(\gamma_0 m_e c)$, K is the conventional undulator parameter), analytical solutions to the single-particle trajectories read:

$$\dot{x}_0 = \upsilon_{\parallel}\frac{\omega_{\perp}\Omega_0}{\Omega_0^2-\Omega_{\parallel}^2}\left[\cos(\Omega_0\tilde{t}) - \cos(\Omega_{\parallel}\tilde{t})\right],$$
$$\dot{y}_0 = \upsilon_{\parallel}\frac{\omega_{\perp}\Omega_0}{\Omega_0^2-\Omega_{\parallel}^2}\left[\sin(\Omega_{\parallel}\tilde{t}) - \frac{\Omega_{\parallel}}{\Omega_0}\sin(\Omega_0\tilde{t})\right], \qquad (1)$$
$$\dot{z}_0 = \upsilon_{\parallel} + \upsilon_{\parallel}\frac{\omega_{\perp}^2\Omega_0}{\Omega_0^2-\Omega_{\parallel}^2}\left[\frac{\cos([\Omega_0-\Omega_{\parallel}]\tilde{t})}{2(\Omega_0-\Omega_{\parallel})} + \frac{\cos([\Omega_0+\Omega_{\parallel}]\tilde{t})}{2(\Omega_0+\Omega_{\parallel})} - \frac{\cos(2\Omega_0\tilde{t})}{4\Omega_0}\right];$$

where $\upsilon_{\parallel} = \kappa V_{\parallel}$ is the mean velocity of constant motion, $\Omega_0 = \kappa\omega_0$, $\Omega_{\parallel} = \sigma\omega_{\parallel}/\sigma_0$ ($\sigma_0 = \omega_{\parallel}/\omega_0$), and the overdots denote differentiation with respect to the transit time $\tilde{t}$. Here κ and σ are self-consistently defined as functions of ε and σ_0 by the system of algebraic equations

$$\kappa = 1 - \frac{\varepsilon^2(3\kappa^2+\sigma^2)}{4(\kappa^2-\sigma^2)^2}, \sigma = \sigma_0\left[1 + \frac{\varepsilon^2(\kappa^2+\sigma^2)}{4(\kappa^2-\sigma^2)^2}\right].$$

It turns out that out of four quantities λ_w, $B_{\perp}$, $B_{\parallel}$ and $V_{\parallel}$ only two their dimensionless combinations, ε and σ_0, define the electron dynamics.

For $\Omega_{\parallel} \approx \Omega_0$ the amplitude of transversal oscillations in (1) tends to infinity. This signifies a resonant transfer of kinetic energy between longitudinal constant motion and oscillatory degrees of freedom (*magnetoresonance*). However, analytical solutions (1) are not valid for all values of ε and σ_0. It turns out that for a some connected set of system parameters centered on the magnetoresonance and shown in Fig. 1 in black, there occurs chaotic behavior. The Chirikov resonance-overlap criterion[6] provides a characterization of the chaotic parameter set

$$|\Omega_0 - \Omega_{\parallel}| < \omega_{\perp} \quad (|\kappa - \sigma| < \varepsilon). \qquad (2)$$

In article[5] for the zones of regular dynamics, we found the weakly-relativistic dispersion equation of a hybrid planar FEL under the undulator synchronism $\omega - k_z^0\upsilon_{\parallel} = \Omega_0$:

$$\delta k_z(\delta k_z - \frac{\delta\omega}{\upsilon_{\parallel}})^2 = -\frac{\pi|I_0|}{2\gamma_0^3c^2\overline{U}}\left(\frac{\omega_{\perp}\Omega_{\parallel}}{\Omega_0^2-\Omega_{\parallel}^2}\right)^2\frac{\omega}{ab}\sin^2\left(\frac{\pi\bar{x}}{a}\right).$$

Here $\delta\omega = \omega - k_z^0\upsilon_{\parallel} - \Omega_0$ is a small mismatch from the ideal synchronism; $\overline{U} = m_e\overline{\upsilon}_{\parallel}^2/(2|e|)$. The gain, G, is defined as $8.63\,\mathrm{Im}\,\delta k_z$; while $\mathrm{Re}\,\delta k_z$ provides a correction to the 'cold'

978-1-4244-2120-6/08/$25.00 ©IEEE.

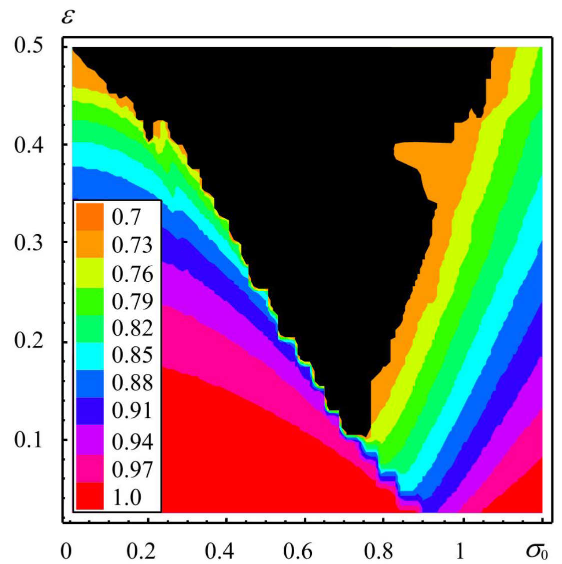

Fig. 1. Contour plot of κ as function of dimensionless coupling ε and guide magnetic field σ_0. The zone of chaotic dynamics is given in black; color marks two zones (left and right) of regular dynamics.

propagation constant, k_z^0, caused by presence of the electron beam. The Chirikov resonance-overlap criterion (2) provides an estimate for the *maximal* (near) *magnetoresonant gain* of a hybrid planar FEL:

$$G_{res}^{\max} \approx \frac{3.7}{\gamma_0}\left(\frac{\pi\omega |I_0|}{abc^2\bar{U}}\right)^{1/3} \sin^{2/3}\left(\frac{\pi\bar{x}}{a}\right);$$

It is attainable in the zone of regular dynamics *above* the magnetoresonance ($B_{\|} > B_{\|}^{res}$).

Results of calculations for the frequency f and gain G are presented in Fig. 2. The solid lines represent analytically obtained quantities while the dots correspond to those calculated numerically. One can see that there are two regimes for pumping kinetic energy into transversal degrees of freedom of a hybrid planar FEL: the first one (advocated here as more, if not the only, efficient one for the terahertz waveband) consists in low amplitude of the undulator magnetic field and the guide magnetic field as close to its magnetoresonant value as only consistent with the regular dynamics; the second regime (usually considered in the literature) would also provide the same magnetoresonant multiplier $\omega_{\perp}/(\Omega_0 - \Omega_{\|})$ through a high absolute value of $|\Omega_0 - \Omega_{\|}|$ given by an off-resonant guide magnetic field $B_{\|}$ and a necessarily high value of the amplitude of undulator magnetic field $B_{\perp}$. It, however, seems that values of $B_{\perp}$ in excess of 10 kG are currently practically unattainable while guide magnetic fields of up to 50 kG can be presently achieved.

We also analytically find that operation just above the magnetoresonance in the zone of regular dynamics provides not only greater values of gain and efficiency but that the gain is independent of spatial amplitude of undulator magnetic field $B_{\perp}$ (cf. Fig. 2 in[7]). Besides, it is less sensitive to space-charge effects. These results suggest that a hybrid planar FEL can successfully provide powerful microwave radiation in the terahertz gap, i.e. in the spectrum segment from 0.3 to 3 THz.

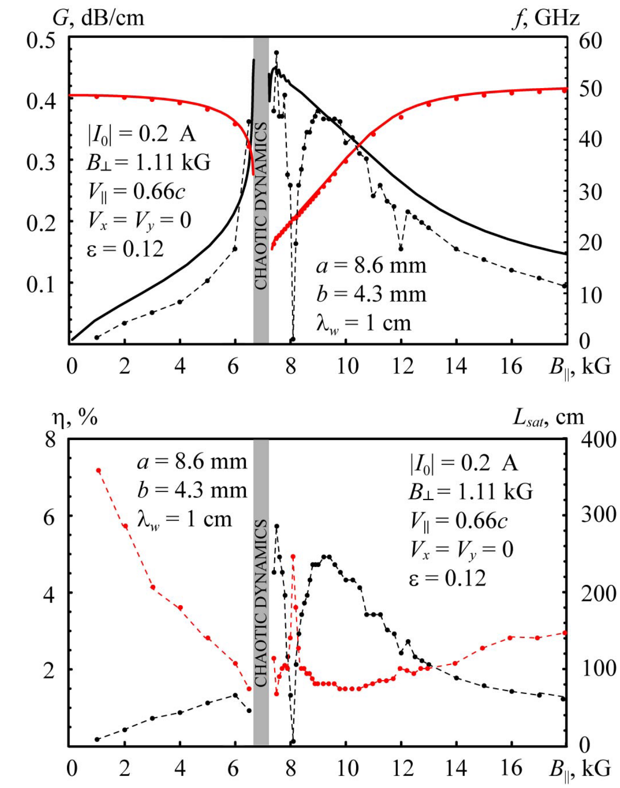

Fig. 2. Top: gain G and frequency f; bottom: efficiency η and saturation length L_{sat}. Solid black and red lines are obtained from analytical expressions; the dots of respective colors united by dashed lines are results of direct numerical simulation for a planar regular waveguide of dimensions $a \times b$; V_x, V_y and $V_{\|}$ are the x-, y- and z-components of the beam velocity at the entrance to the interaction region ($z = 0$); λ_w and I_0 are the undulator spatial period and beam current, respectively. All quantities are calculated as functions of $B_{\|}$ for the TE_{10} mode amplification under the undulator synchronism. The darkened area corresponds to the analytically calculated zones of chaotic dynamics around the magnetoresonance $B_{\|}^{res} \approx 6.67$ kG.

REFERENCES

[1] P. Sprangle and V. L. Granatstein, "Enhanced gain of a free-electron laser", *Phys. Rev. A*, vol. 17, pp. 1792–1793, 1978.

[2] K. Sakamoto, T. Kobayashi, Y. Kishimoto, S. Kawasaki, S. Musyoki, A. Watanabe, M. Takahashi, H. Ishizuka, and M. Shiho, "Beam divergence with harmonic gyroresonance in focusing wiggler and axial field", *Phys. Rev. Lett.*, vol. 70, pp. 441–444, 1993.

[3] B. P. Yefimov, K. V. Ilyenko, T. Yu. Yatsenko, and V. A. Goryashko, "Theoretical model and experimental observation of resonance generation of free electron laser", *Telecommun. Radio Engineer.*, vol. 61, pp. 243–255, 2004.

[4] V. A. Goryashko, K. V. Ilyenko, and A. N. Opanasenko, "Renormalized synchronism conditions for a hybrid weakly-relativistic waveguide free-electron laser", *Telecommun. Radio Engineer.*, vol. 65, pp. 991–1003, 2006.

[5] V. A. Goryashko, K. V. Ilyenko, and A. N. Opanasenko, "Dispersion relation for a hybrid planar ubitron", *Radiophys. Electron.*, vol. 11, pp. 440–446, 2006. (in Russian)

[6] B. V. Chirikov, "Universal instability of many-dimensional oscillator systems", *Phys. Rep.*, vol. 52, pp. 263–379, 1979.

[7] N. S. Ginzburg, R. M. Rozental, N. Yu. Peskov, A. V. Arzhannikov, and S. L. Sinitskii, "The simulation of a free-electron laser amplifier with a ribbon relativistic electron beam", *Tech. Phys.*, vol. 46, pp. 1545–1548, 2001.

978-1-4244-2120-6/08/$25.00 ©IEEE.