

# Effective Permittivity and Scattering Model for the Evaluation of the Leaf Water Status

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**Abstract**—An electromagnetic model of plant leaves at THz frequencies is presented. It allows for the determination of the leaf water status. The model bases on an effective medium theory and extends the model by Landau, Lifshitz, and Looyenga. The influence of scattering, which is dominant at higher frequencies, is described by a Rayleigh roughness factor. Results of THz time-domain spectroscopy measurements on *Coffea arabica* leaves are used to validate the model.

## I. INTRODUCTION AND BACKGROUND

THE determination of the leaf water status is of high importance for farmers and plant physiologists. It can provide valuable information in irrigation management and helps to control drought stress. The state of the art techniques in determining the leaf water content are either destructive methods or work only in reflection geometry like near-infrared spectroscopy.

Mittelman et al. applied in 1996 [1] for the first time THz radiation to study the water content in leaf tissue. Later Hadjiloucas et al. [2] developed a first model for the absorption of plant leaves with varying water content. Scattering losses were neglected in this approach, because the measured data were restricted to frequencies below 500 GHz. Furthermore, this model does not describe the refractive index, which is a key parameter for a coherent detection scheme. Here, we present a model which allows for the calculation of the absorption  $\alpha$  and the refractive index  $n$ . To determine the water content of a leaf, either  $n$  or  $\alpha$  can be used. Therefore, our model is completely independent of any measurement technique and works for investigations with pulsed as well as with continuous wave THz radiation. Additionally, we included scattering losses in our model, which have crucial influence on the absorption especially at higher frequencies above 1 THz.

## II. THEORY

Considering a leaf as a multi compound mixture, we conclude, that it mainly consists of the following three parts: air, water, and the solid plant material. Therefore, an effective medium theory (EMT), which describes the resulting material properties of a mixture, is in principal suitable to model the dielectric constant of this biological sample.

A widely-used EMT is the Landau, Lifshitz, Looyenga model (LLL) [3], which characterizes mixtures consisting of irregular shaped particles. However, the LLL model is just capable to describe the effective permittivity of a compound consisting of two parts. Thus, we provide a third-order extension of the LLL model to account for the three parts of

the leaf:

$$\sqrt[3]{\epsilon_L(f)} = \xi_W \sqrt[3]{\epsilon_W(f)} + \xi_S \sqrt[3]{\epsilon_S(f)} + \xi_A \sqrt[3]{\epsilon_A(f)},$$

while  $\xi_W + \xi_S + \xi_A = 1$ ,

where  $\xi$  is the concentration of the components. The index  $L$ ,  $W$ ,  $S$ ,  $A$  refers to the leaf, the water, the solid plant material, and the air, respectively.

The resulting complex permittivity already includes the absorption losses. However, the scattering at the leaf's faces is neglected by this simple approach. To overcome this limitation, we summarize the two contributions, absorption and scattering losses, to determine the total absorption coefficient:

$$\alpha_{res} = \alpha_{abs} + \alpha_{scat}.$$

The influence of scattering due to surface roughness can be described by a Rayleigh roughness factor [4]. To account for the frequency varying reflectivity, we additionally take the contrast in permittivity  $\Delta\epsilon$  into account, which leads to the scattering caused quasi absorption:

$$\alpha_{scat} = \left( \Delta\epsilon(f) \cdot \frac{4\pi\tau \cos(\theta)}{\lambda T} \right)^2,$$

with  $\Delta\epsilon(f) = \sqrt{\epsilon_L(f)} - 1$ .

Here,  $\tau$  denotes the degree of surface roughness,  $\theta$  is the angle of incident,  $\lambda$  the free space wavelength, and  $T$  the thickness of the leaf.

## III. MEASUREMENTS

A series of fresh and water stressed *Coffea arabica* leaves were measured with a standard THz time-domain spectrometer (TDS) in transmission geometry. Afterwards the leaves were detached from the plant, the thickness and mass was measured and the water was pressed out. With information on the thickness  $T$  and the mass  $M$  of each leaf before and after pressing the concentration  $\xi$  of water, air, and solid plant material can be determined by

$$\xi_L = \frac{T_P}{T_F}, \quad \xi_W = \frac{M_F - M_P}{(T_F - T_P)\rho_W A}, \quad \xi_A = 1 - \xi_L - \xi_W,$$

where  $\rho_W$  is the specific gravity of water and  $A$  the leaf's area. The indices  $F$  and  $P$  denote the measured values before and after pressing, respectively.

We derived the pure water permittivity by a dual Debye model analogue to [5]. The dielectric properties of the solid plant material were determined by TDS measurements on the pressed and dried leaves.

The measured refractive index and the absorption of two leaves, a fresh and a stressed one, are shown in (Fig. 1) together with the models prediction. While the refractive index reveals an intense anormal dispersion, the absorption coefficient increases steadily with the frequency. Due to the high permittivity of water, the refractive index and the absorption are larger for fresh than for water stressed leaves.

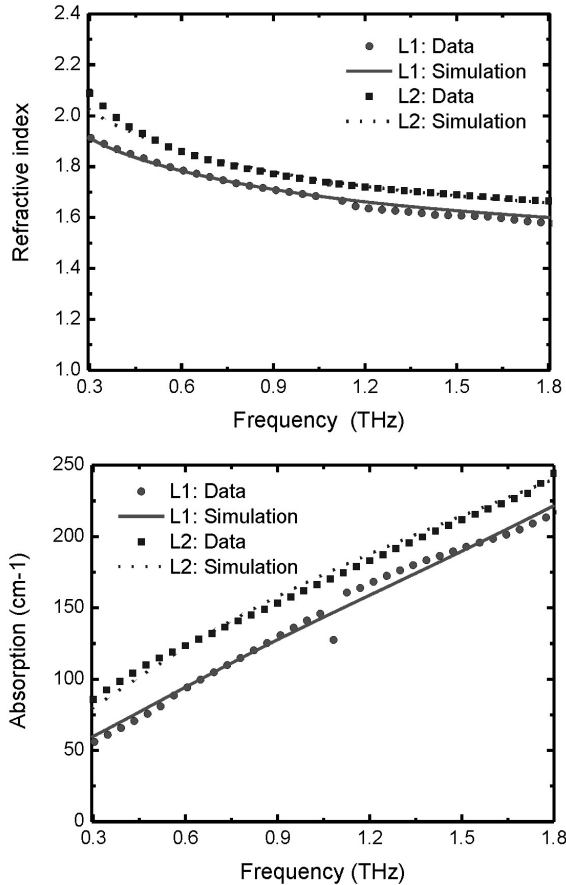


Fig. 1: Measured and simulated refractive index and absorption of fresh (L2) and stressed (L1) *Coffea arabica* leaves.

The models prediction is in good agreement with the measured values in the complete frequency range between 0.3 and 1.8 THz.

#### IV. CONCLUSION

Combining a third order EMT with a surface scattering formulism, we derive a model, which is capable to determine the refractive index and the absorption of plant leaves as a function of the volumetric water content. Despite the fact that the model is validated by a comparison with THz TDS measurement data in the frequency range between 300 GHz and 1.8 THz, it is in principle applicable to any spectral region where the specific material permittivities are known. Therefore, the model may be a used in the development of novel measurement techniques for the water status of vegetation.

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