Gyrotron Mode Competition Calculations: Investigations on the Choice of Numerical Parameters

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***Abstract*—The proper choice of numerical parameters in mode competition calculations for gyrotron cavities, like time step size, azimuthal phase discretisation and selection of included modes, is discussed. As a first step, simple rules are derived and justified.**

I. Introduction and Background

CURRENT megawatt class gyrotrons for fusion plasma heating have reached not only high power, but also a very high order of operating mode[1,2,3]. Thus mode competition calculations for gyrotron interaction get increasingly complex, and the proper choice of numerical parameters is not at all obvious. This problem will be discussed below in the frame of slow variable codes for time-dependent, self-consistent multi-mode calculations, like the FZK SELFT code[4].

At mode eigenvalues well above 45 (corresponding to the well established $TE_{22,6}$ mode), the mode spectra of gyrotron cavities get quite dense, which means the frequency separation between competing modes decreases and the number of modes to be taken into account in a calculation increases. First question must be which modes should be included in the calculation. The time step size will depend on the included modes. This is discussed on the example of the behavior of radial satellite modes $TE_{m,p\pm1}$. Finally, discretisation over azimuth is a major issue, while the discretisation along the *z*-axis and the number of macroparticles used appears to be less critical according to our test cases.

II. Discussion

When selecting modes to be included in mode competition calculations, obvious and well known rules are to choose modes with high electron beam coupling and within a limited bandwidth around the cyclotron frequency of the electron beam. Our investigations indicate that the coupling limit may be as high as 70 % of the operating mode, since modes operating in gyrotron interaction tend to suppress other modes with even only slightly less coupling. Speaking for the bandwidth of included modes, parameter changes during the calculation must be considered. Typically, in gyrotron startup simulations the cyclotron frequency decreases with increasing voltage. Therefore the frequency band must be expanded from some usual ±5 % around the working mode to rather -5 % / +20 %.

Having determined the list of modes, it is a save choice to set the time step size to less than half of the inverse bandwidth. The time step may be further increased, but it must at least be small enough to describe beating frequencies between any active modes. Excellent examples for correct and too large choices of time step size are simulations which include radial satellite modes $TE_{m,p\pm1}$ (Fig. 1a). These modes can draw energy from the electron beam which is bunched by the operating mode $TE_{m,p}$, because they have the same azimuthal structure – but only if their frequency is pulled to the operating modes' frequency, as in Fig. 1a. It is also instructive to look at the field profiles in this case (Fig. 2), which reflect the fact that the satellite modes are not excited at their resonant frequency. In contrary, with a too large time step size of 0.05 ns (Fig. 1b), this frequency pulling can not be modeled. The radial satellites can not accumulate energy and decay over time. It is typical for such simulations that the energy conservation is violated, too.

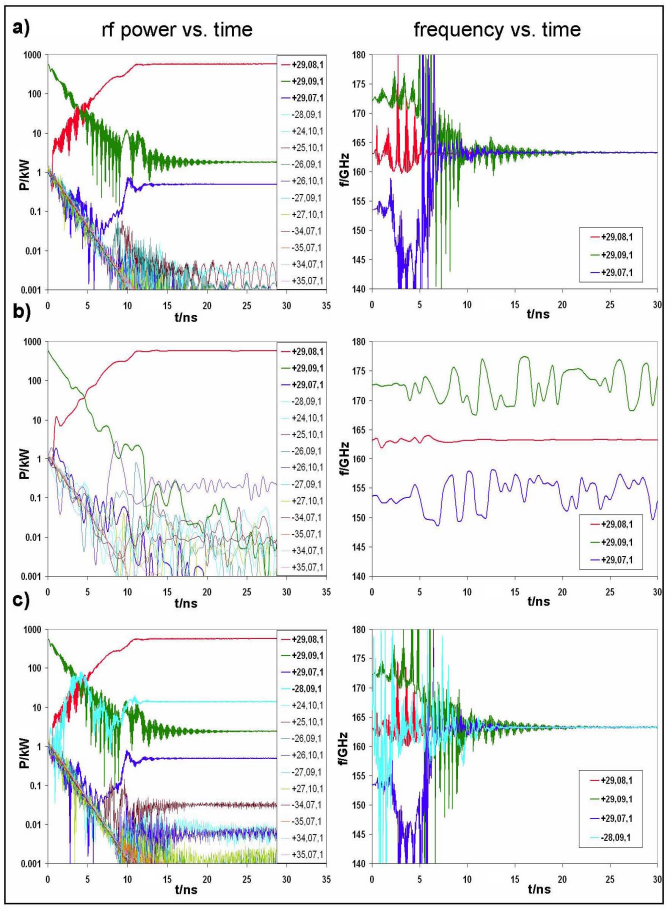

Figure 1: Power vs. time (left) and frequency vs. time (right) for an arbitrary simulation example: operating mode $TE_{+29,8}$ (red), radial satellite modes $TE_{+29,7}$ (blue) and $TE_{+29,9}$ (green); a) correct calculation: time step size 0.01ns, b) incorrect calculation: time step size 0.05ns, c) incorrect calculation: 19 equidistant phase averaging steps.

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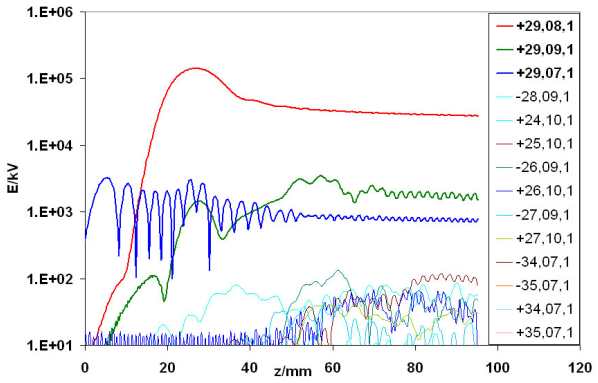

Figure 2: Field profiles vs. z for the modes in Fig. 1a. $TE_{+29,7}$ (blue) is above cutoff in the downtaper, $TE_{+29,9}$ (green) is below cutoff even in the cavity. The latter mode mainly draws energy from the beam bunched by the operating $TE_{+29,8}$ (red) in the cavity uptaper ($z > 40$ mm). The lower satellite mode $TE_{+29,7}$ is operating far from its cutoff, so it is not in phase with the electron beam and exchanges energy with the beam periodically along z.

Next topic is the choice of azimuthal discretisation of the electron beam, which, depending on the employed formalism, may appear as an azimuthal phase averaging among modes. The simulation model accounts for the azimuthal structure of each mode $TE_{m,p}$ analytically. But in multi-mode calculations the phase difference between two modes with different m changes over the azimuth angle φ. Since these beating phases repeat itself over φ, it is sufficient and common practice to use only a few discretisation steps n_φ, in order to cover a representative sample of phase differences, but not necessarily to fully describe the modes' structures over φ. The drawback is that the few chosen sample points must be a representative sample of phase differences between any pair of modes with different m, which gets increasingly difficult with many modes included. Whenever a pair of modes with azimuthal indices $m_1 \neq m_2$ fulfills the condition $(m_1 - m_2) / n_\varphi = integer$, an equidistant number of n_φ azimuthal steps will calculate identical azimuthal phase differences. This means there is no averaging over azimuth effective between these modes. An obvious example is to set $n_\varphi = m$ and include both rotation directions of a mode $TE_{m,p}$ and $TE_{-m,p}$, which makes these two modes couple strongly. Figure 1c shows another example: Here the $TE_{-28,7}$ mode is coupled to the operating mode $TE_{+29,8}$ artificially by $n_\varphi = 19 = (29 - (-28)) / 3$. The frequency of $TE_{-28,7}$ is also pulled to the operating modes' frequency. In the correct simulation in Fig. 1a the $TE_{-28,7}$ mode decays as it should.

By checking the given condition such cases can be avoided; it is desirable to end up with $(m_1 - m_2) / n_\varphi$ being a fraction number with denominator higher than 5. In any case n_φ can be set to the next higher prime number above $2m_{max}$, which may be large, but still significantly lower than $10m_{max}$. The latter would be a usual choice for complete discretisation over φ, which, of course, would be required when the simulated structure is not symmetric over azimuth any more.

While correct time step size and azimuthal discretisation is crucial, as shown above, two other quantities appear less critical: For z the usual rule of thumb for numerics seems to be sufficient: Use $1/10^{th}$ or less of the free space wavelength. The number of macroparticles over slow variables phase is critical in stationary simulations[5], but turns out to be much less sensitive in time-dependent simulations. The bunching process, which collects the electrons or macroparticles in a small range of slow variables phase, is highly desired, but leaves only a few particles in a broad region of phase. Particles remaining in such rarified ranges are not representative any more, but can have strong influence depending on their individual starting position. This effect leads to recognizable deviations in stationary calculations, but is balanced in non-stationary simulations by the fact that over time the influence of such particles is averaged out. Our tests show that effects caused by insufficient particle numbers vanish in non-stationary calculations even at macroparticle quantities as low as 16, while in stationary calculations even 300 particles may be not enough.

III. Conclusions

The proper choice of simulation parameters is crucial for the validity of high power gyrotron interaction simulations, particularly in highly overmoded cavities. It is in particular important to keep the time step size sufficiently low. The azimuthal discretisation may be reduced from a physically save choice, but then care must be taken to avoid artificial coupling between some modes. In contrast to stationary simulations the number of macroparticles does not need to be very high, because the slow variables entry phase is inherently fine discretized by the required small time step size.

The derived rules are useful to setup both efficient and reliable simulations. Even though the results are discussed in the frame of a slow variables model, they are based on the physical behavior and can therefore be applied to any numerical simulation of highly overmoded gyrotron cavities. Still a perfect reproduction of experimental results may not be expected, taking into account realistic tolerances on the one hand, and remaining numerical uncertainties on the other hand. However, obeying the discussed rules should at least make sure that the simulated results represent reasonable solutions that could be obtained in a real system. These investigations will be expanded to estimate the influence of tolerances, in order to gain the design safety required by current high power gyrotron projects.

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