Numerical Analysis of Complex Mirror Transmission Lines

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***Abstract*—A versatile tool for the numerical analysis of complex quasi-optical transmission lines for millimeter waves is presented. The object-oriented design allows interactive numerical investigations, iterative mirror synthesis and phase reconstruction from amplitude measurements. The transmission line of the W7-X Stellarator in Greifswald with more than 100 mirrors and a complicated geometry is used as an example, in which the microwave beams can be numerically modeled from the gyrotron cavities up to the plasma in the torus.**

I. Introduction

QUASI-OPTICAL transmission lines for electron cyclotron resonance heating (ECRH) in fusion devices are an attractive alternative to commonly used HE_{11}-waveguide transmission lines. Low transmission losses, maintainability and low mode conversion are some of their features. They are commonly designed on the basis of the Gaussian mode theory and ABCD matrix calculus. In large ECRH plants, however, these transmission lines have a complex 3D geometry and many mirrors. This complicates the numerical analysis with real input beams. In this paper, the transmission lines of W7-X will be used as a practical example for the modeling of such systems. It extends over more than 50m and transmits 10 gyrotron output beams. Due to the fact that the largest part of the transmission line is a so-called multiple-beam waveguide [1], it has "only" some 120+ mirrors. The fact that it was possible to adapt the building to the transmission line requirements was also helpful in reducing the number of mirrors.

The modeling of such a large and complicated mirror setup is difficult to handle for traditional codes, which usually make use of the plane wave decomposition in conjunction with the FFT to propagate paraxial beams. In addition, the plane wave decomposition is not very accurate for mirrors with a large tilt angle, as it only operates apertures which are perpendicular to the optical axis. Classical EM codes can not be used as the structures are too large compared to the wavelength where geometrical optics or Fraunhofer diffraction are too inaccurate.

II. Mathematical Background

The field propagator which is used can be seen as a generalized plane wave decomposition and solves the scalar Helmholtz equation in the wavenumber domain. It (back)propagates the field between two plane apertures with arbitrary orientation in a global coordinate system. An aperture is represented by the $z=0$ plane of its local coordinate system. If the position of the target has a positive z-component in the local coordinate system of the source and the position of the source has a negative z-component in the local coordinate system of the target, then a forward propagation is performed. In the opposite case, a backward propagation is carried out. Any other case makes no sense, i.e. all plane waves pass an aperture from the negative-z half space to the positive-z half space. So there is only one operator which distinguishes automatically between forward and backward propagation [2]:

$$U_t(\vec{k}_{x,y})=\frac{1}{\vec{k}_z}U_s(\bar{S}^T\bar{T}\vec{k}_{x,y})r(\bar{S}^T\bar{T}\vec{k}_z)e^{-j\bar{T}\vec{k}_z d}$$

The subscripts of vectors refer to their respective components, U_t and U_s are the two-dimensional target and source distributions in the wavenumber domain, r is the ramp function, d is the distance between the apertures, the columns of the dyads $\bar{S}$ and $\bar{T}$ consist of the base vectors of the local coordinate systems of the source and target aperture within the global coordinate system and $\vec{k}_z=\sqrt{k_0^2-\vec{k}_x^2-\vec{k}_y^2}$. The transformation from the spatial domain to the wavenumber domain is carried out via the FFT.

The electric vector potential in a plane aperture $z=0$ is represented by a constant complex vector $\vec{F}_0$ and a two-dimensional scalar field u: $\vec{F}(x,y,0)=u(x,y)\vec{F}_0$. As the ratio of the components of $\vec{F}_0$ is fixed, we need to solve the wave equation only once for the scalar u. If we took $\vec{E}_0$ or $\vec{H}_0$ instead of $\vec{F}_0$, a scalar code would not work because $\vec{E}$ and $\vec{H}$ must also obey the Maxwell equations and hence, the ratio of their components is not constant. Only in the case of individual waveguide modes or plane waves (free space "modes") we can derive all other field components from the scalar solution of a constant $\vec{E}_0$ or $\vec{H}_0$ (e.g. $\vec{E}_z$ or $\vec{H}_z$ in waveguides). $\vec{E}$ and $\vec{H}$ in the aperture can be calculated from $\vec{F}(x,y,0)$. As it is complex, $\vec{F}_0$ can represent any polarization state of the wave beam. The differential operators for the calculation of $\vec{E}$ and $\vec{H}$ from $\vec{F}$ can be conveniently applied in the wavenumber domain (multiplication instead of differentiation). The boundary conditions for $\vec{F}$ on a perfect electric conductor (a mirror) are contrary to the boundary conditions for $\vec{E}$: the normal components of the incident and reflected wave cancel out and the tangential components add up. Here, a plane mirror is assumed although it might be a flat mirror (corresponding to a thin lens). For the investigation of the generated cross-polarization it would be necessary to split the reflected beam into three beams corresponding to the individual components of $\vec{F}_0$. However, in the case of a flat mirror the cross-polarization is usually negligible. For the reflection from a polarizer, the 3D-Jones matrix from [3] can be used. First, the incident field $\vec{E}_{0i}$ of the beam center is obtained from $\vec{F}_{0i}$, then it is multiplied by the Jones matrix and finally the reflected $\vec{F}_{0r}$ is calculated from $\vec{E}_{0r}$. $\vec{F}_{0r}$ has a unique solution if $\vec{F}_{0r}\cdot\vec{k}_{0r}=0$ is imposed, which has the effect that there will be no cross-polarization in the outer regions of a linearly polarized beam ($\vec{k}_{0r}$ is the dominant plane wave of the

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reflected beam).

The initial source distribution can be taken directly from a waveguide output, as TE modes do only have an $\vec{F}_z$ component, or from any EM code if a plane can be found where one component of the incident electric field is negligible compared to the other two components. For a description of how $\vec{F}$ can be obtained from $\vec{E}$, see [4].

III. Implementation

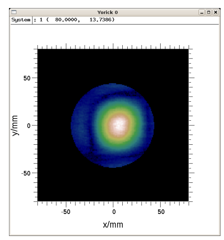
In order to handle the complex 3D geometry and the high number of mirrors, the code is implemented in the form of a high level interpreted object-oriented programming language. This results in a high flexibility which allows for code reuse in the case of similar parts and simple coordinate transforms. Mirrors, polarizers, vacuum windows and the like are instances of a class of type “aperture”. The most important attributes of that class are a complex 2D field for the storage of u with dimensions according to the size of the aperture, a real 2D field for the storage of the power distribution on a window or the phase correction of a flat mirror, a complex 3D vector for the storage of $\vec{F}_0$ and a real 3x3 matrix for the storage of the dyad which maps the local coordinate system to the global one. The most prominent methods of that class are the setup of the attributes, methods for rotation and translation of the apertures, a method to calculate the (back)propagated field on another object of type aperture and a method which we call “toggle” which needs some explanation:

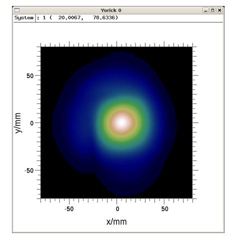
When an aperture serves as the target of a projection, the source must be located in its (local) negative half-space (see above). If it is a mirror, the reflected beam is also directed in the negative half space which makes it impossible to turn that aperture into a source for the next aperture which is also located in the negative half-space. The purpose of the “toggle” operation is to overcome this problem. It turns the aperture by 180° around its y-axis, it sets the orientation dyad accordingly and it exchanges $u(-x)$ and $u(x)$, such that the field distribution remains the same in the global coordinate system. The last operation is to multiply $\vec{F}_{0x}$ by -1 since the x-axis points now in the opposite direction. This is not done for $\vec{F}_{0z}$ because the boundary condition on a perfect electric conductor is automatically fulfilled by the rotation of the z-axis. In the case of a polarizer, the Jones-Matrix must be applied instead. If it is not a plane mirror or polarizer, the corresponding phase corrector must also be applied, but this is not part of the “toggle” operation.

The above methods are complemented by other methods for the calculation of diffraction losses, phase unwrapping or phase correction and the like, such that they form a tool box for all required operations on the optical elements. This serves now as a language to describe a complex quasi-optical system with many elements similar to the textual description of scenes in 3D modelers. Iterative algorithms for phase reconstruction and mirror synthesis or Fox-Li iterations on quasi-optical resonators can be implemented in minutes. Due to the application of the FFT and the fact that only the scalar Helmholtz equation is solved, the interpreter is fast enough to do trial-and-error experiments and to optimize parameters manually.

IV. Practical Applications

With the help of the described code it was possible to model the complete W7-X transmission line from the gyrotron cavity up to the plasma boundary. This involves 19 mirrors and 2 polarizers, where 3 mirrors are internal mirrors of the gyrotron, 7 mirrors are multi-beam mirrors and 2 mirrors belong to the ECRH launcher in the plasma vessel. However, a more realistic scenario is to inject measured beams (measured amplitude + reconstructed phase) into the model to investigate the field pattern and the diffraction losses at any position in the transmission line. Calculations of an injected beam which was measured on the manufacturer's gyrotron test stand have shown that non-Gaussian but paraxial content is not completely lost during the transmission. The figure below shows that the side lobes in the gyrotron window (left) can still be recognized at the torus window (right).





However, this is not an undesired behavior as the non-Gaussian content is still delivered to the plasma instead of being lost in the transmission line. The similarity of the figures is also a confirmation for the imaging properties of the quasi-confocal design of the system, yielding broadband transmission characteristics.

Another possible application would be the in-situ beam reconstruction where the field at any position can be calculated from infrared images of any three (not necessarily consecutive) mirror surfaces. As the IR camera looks perpendicularly onto the mirror it will not be hit by the microwave beam. This can be helpful in geometrically constrained areas and could be programmed in minutes. Note that there is no rectification of the infrared images required (it is even forbidden).

The actual purpose of this development is the calculation of look-up tables for the control of the ECRH launchers of W7-X in conjunction with the polarizers such that the desired content of O- and X-mode polarization is produced, depending on the poloidal and toroidal launching angles. Other applications are the investigation of tolerances or mechanical drifts in the transmission line or to identify locations where the given beam pattern differs too much from the design beam (especially on vacuum windows).

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