# Numerical Optimization of Smooth-Wall and Corrugated Horn Antennas

Burkhard M. Plaum
Institut für Plasmaforschung, Pfaffenwaldring 31, D-70569 Stuttgart, Germany

***Abstract*—A method for optimization of smooth-wall or corrugated cylindrical waveguide antennas is presented. It uses a hybrid simulated annealing/downhill simplex method to find a taper geometry, which will convert the incoming waveguide mode into the desired pattern. The method is very flexible with respect to the definition of the optimization goal, which can, in addition to the desired aperture field, also include geometrical constraints and low ohmic losses.**

## I. Introduction And Background

Waveguide antennas are used in many applications ranging from low power systems like plasma diagnostics to MW level ECRH facilities. Depending on the application, important characteristics are small side lobes, low cross-polarization or a focusing behavior. The presented method takes arbitrary aperture fields, which are expanded into amplitudes of waveguide modes. The optimizer then tries to find a geometry function *r(z)*, which converts the incoming waveguide mode into the desired aperture spectrum.

## II. Algorithm

For the optimization, a hybrid *simulated annealing / downhill simplex* method is used. In earlier works[1] it showed good behavior even for difficult optimization problems. The geometry function *r(z)* is given in terms of Chebyshev polynomials, which allow a very precise approximation of all typical taper geometries and have a good convergence behavior. The two lowest order coefficients are not passed to the optimizer. Instead, they are calculated from the higher order coefficients in order to fulfill the boundary conditions of the input and aperture radii.

One advantage if this optimization method is its flexibility since it can work with different methods for calculating the mode conversion (scattering matrix method or coupled mode equation). Furthermore, one can freely define the rating function denoting the quality of a converter during the optimization. In normal cases, this rating function will be calculated from the overlap integral of the actual aperture field and the desired aperture field. It is, however, possible to include geometrical limits like minimum and maximum values for the converter radius and the slope angle of the wall in axial direction. A maximum acceptable power limit for higher order modes inside the converter, low cross polarisation, ohmic losses or reflections could in principle be included as well. This would, however, require more computation time.

## III. Smooth-Wall Horn Antenna

As a first example, a smooth wall horn antenna, which is fed by a $TE_{11}$ mode and radiates a Gaussian beam at 160.28 GHz, was optimized. Such antennas have the advantage, that they are easier to manufacture, since they have no corrugation. Due to the higher wall losses (in comparison to corrugated components), however, their application is limited to low power. Given parameters were the input and output diameter (4 mm and 18 mm) and the Gaussian waist radius of 4.5 mm. The length was kept fixed at 100 mm. The mode conversion was calculated with a scattering matrix algorithm. Fig. 1 shows the optimized geometry. Fig. 2 shows the aperture field. In co-polarization one can see a Gaussian pattern (Fig. 2, left) with a plane phase-front (Fig. 2, middle).

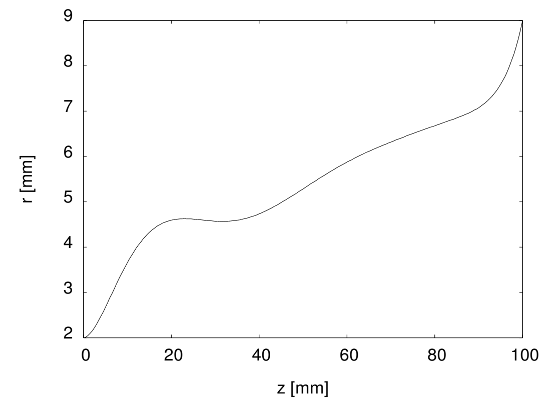


Fig. 1: Geometry function of the optimized antenna

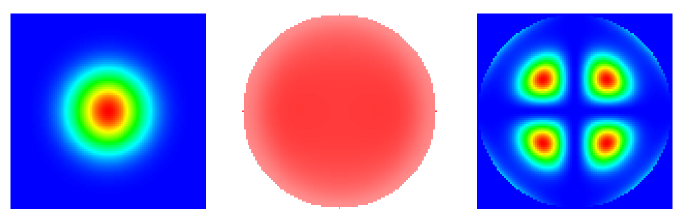

Fig. 2: Aperture field of the optimized antenna. The left and middle images show the amplitude and phase of the co-polarization, the right image shows the amplitude of the cross-polarization. The scale is always linear, difference of maxima in co- and cross-polarization is 29.4 dB.

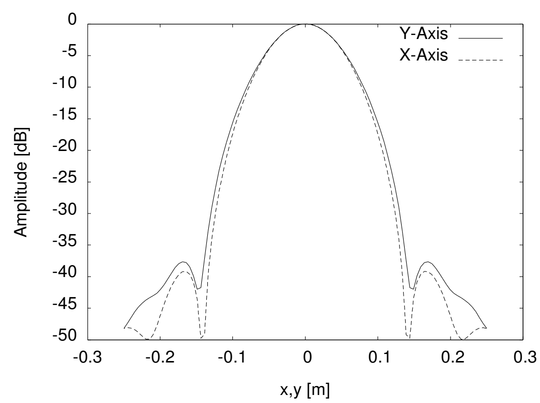


Fig. 3: Far field of the optimized antenna in logarithmic scale. The side-lobe suppression is -37 dB.

The peak field in the cross-polarization (Fig. 2, right) is 29.4 dB below the peak field for co-polarization. Fig. 3 shows

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the far-field profiles in horizontal and vertical direction in logarithmic scale. The distance is 0.5 m. Here, the side-lobe suppression is about -37 dB.

## IV. Corrugated Horn Antenna

The second example is a corrugated horn-antenna, which is fed by an $HE_{11}$ mode and radiates a Gaussian beam. An open-ended $HE_{11}$ waveguide can already be used as a Gaussian beam antenna with 98% mode purity. The remaining power shows up in side-lobes and increases the stray radiation level. In high power applications, a better matching is achieved by generating a spectrum of $HE_{1n}$ modes. In the application for this antenna, the free space beam is directed onto a focusing mirror. This means, that the horn can radiate a slightly defocussing beam, which is compensated by the mirror and allows a shorter converter length. For the given data (Frequency: 140 GHz, input diameter: 87 mm, output diameter: 123.4 mm, length: 546 mm, waist radius: 28.8 mm, waist location inside the waveguide 340 mm from the aperture), the corresponding mode spectrum (i.e. the optimization goal) is shown in Table 1.

| Mode | Power (goal) | Phase (goal) | Power (result) | Phase (result) |
|---|---|---|---|---|
| $HE_{11}$ | 86.18 % | 0° | 86.03 % | 0° |
| $HE_{12}$ | 13.67 % | -22.26° | 13.76 % | -23.57° |
| $HE_{13}$ | 0.14 % | -55.33° | 0.021 % | 151.09° |
| $HE_{14}$ | 0.0038 % | -111.63° | 0.169 % | 39.48° |
| $HE_{15}$ | 0.0018 % | 66.24° | 0.0024 % | 107.81° |
| $HE_{16}$ | 0.0015 % | 116.48° | 0.0163 % | 66.00° |

Table 1: Mode spectrum in the aperture expanded from the Gaussian beam (goal) and produced by the optimized antenna (result).

An analytical solution is a parabola shape horn[2], which has an efficiency of 99.64%. The optimization was done with the same geometrical constraints as for the parabola horn. The mode conversion was calculated by integrating the coupled mode equation. This method is significantly faster than the scattering matrix method, but it works only if all dominant mode are far above cutoff, and the geometry variations are small enough to neglect reflections. Fig. 4 shows the geometries of both the parabola type and the optimized antenna. Fig. 5 shows the amplitude and phase of the aperture field. One can see the Gaussian amplitude pattern and the curved phase front. Since the dominant modes are low-order $HE_{1n}$ modes, the field in cross-polarization is negligible. The optimized solution has an efficiency of 99.74%.

The goal spectrum in Table 1 contains small fractions of higher order modes (e.g. $HE_{13}$), whose phases must be matched for a perfect result. Since phase matching of different modes can only be achieved by a propagation delay, a further optimization would require to allow the optimizer to change the length within wide limits. For the $HE_{11}$ and $HE_{12}$ modes one can see, however, that the goal and the optimization result agree very well.

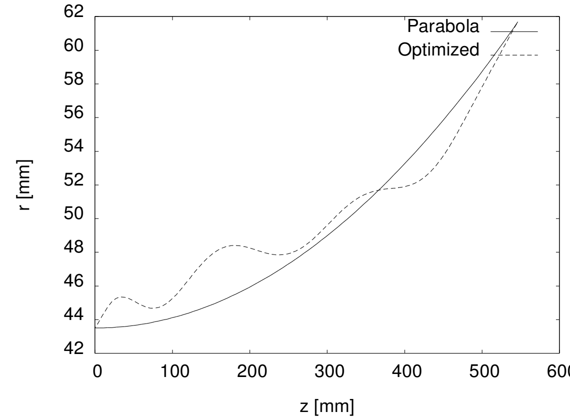


Fig. 4: Geometries of the parabola horn and the optimized solution

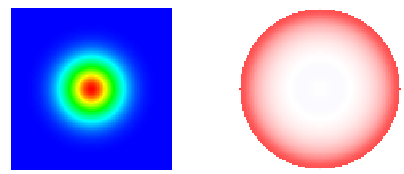

Fig. 5: Amplitude (left) and phase (right) of the aperture field.

## V. Conclusion

The presented algorithm produces optimized solutions, which are always better than conventional ones. Since the optimization goal for horn antennas consists of a complex multimode spectrum, the optimization is computationally more expensive than for waveguide bends, where the goal is maximum power in a single mode with arbitrary phase. In contrast to other methods[3,4], the optimizer has no knowledge of the involved physics since the optimization is done by searching for a global minimum of a rating function in a multidimensional parameter space. On the other hand, the function to be minimized can be freely defined, allowing almost arbitrary optimization goals.

One crucial parameter is the ratio of the aperture size and the beam waist size. Larger values result in increased power in higher order modes, making the conversion from the fundamental mode more difficult. Smaller values result in an increased mismatch due to the truncation of the Gaussian pattern by the limited aperture.

A possible extension would be to keep the wanted beam parameters fixed, but allow the optimizer to change the aperture size. This would, however, significantly increase the optimization time because in addition to the mode conversion, the code must numerically calculate the overlap integral of the wanted Gaussian beam and the aperture field for each iteration.

## References

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